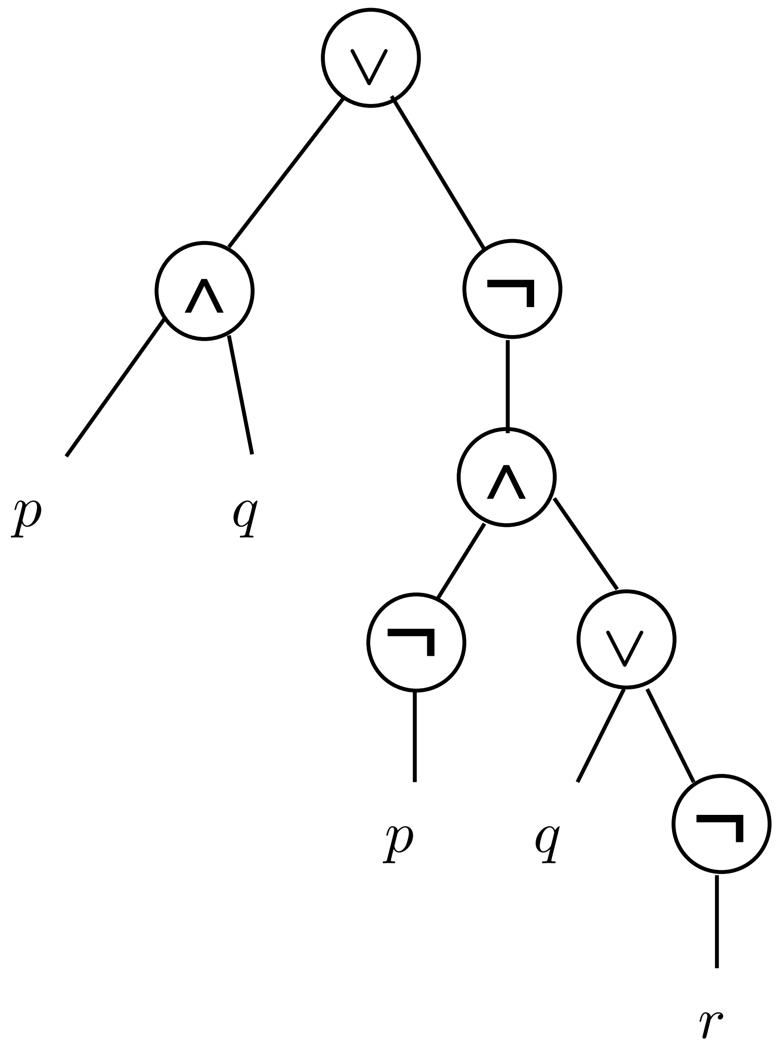
Tseitin Transformation

Let F be $(p \wedge q) \vee \neg(\neg p \wedge (q \vee \neg r))$



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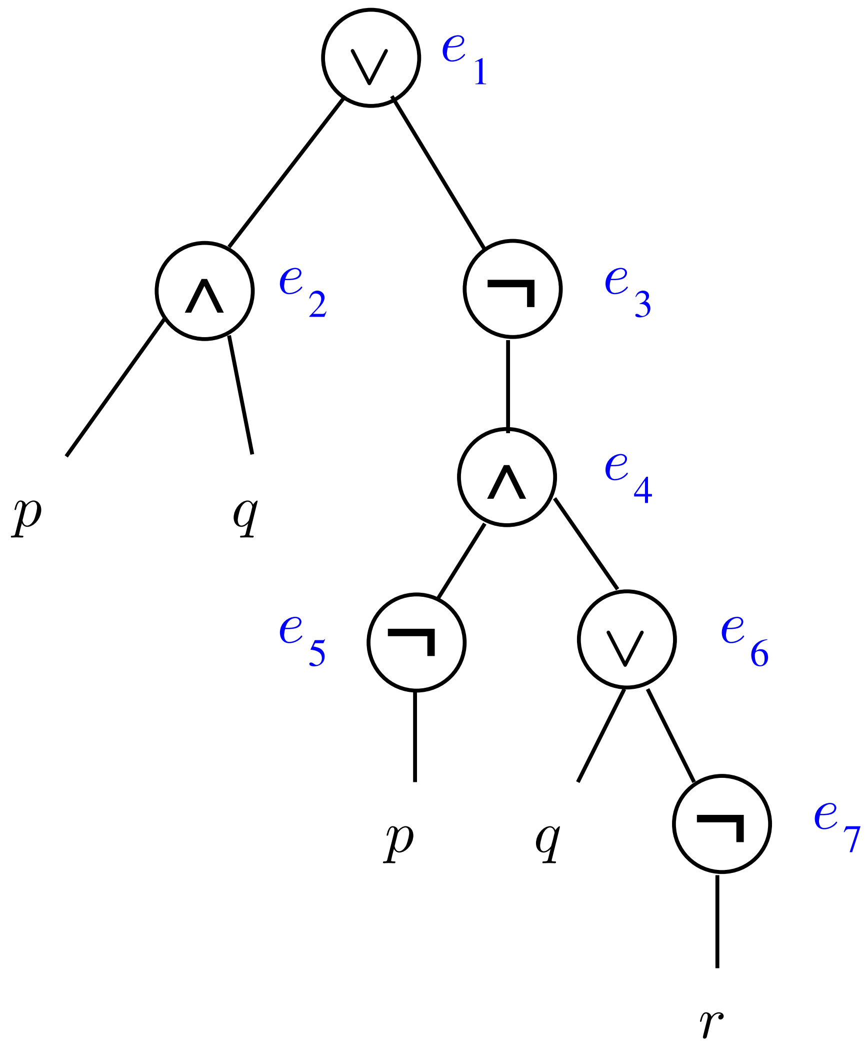
Let F be $(p \wedge q) \vee \neg(\ \neg p \wedge (q \vee \neg r)\ )$



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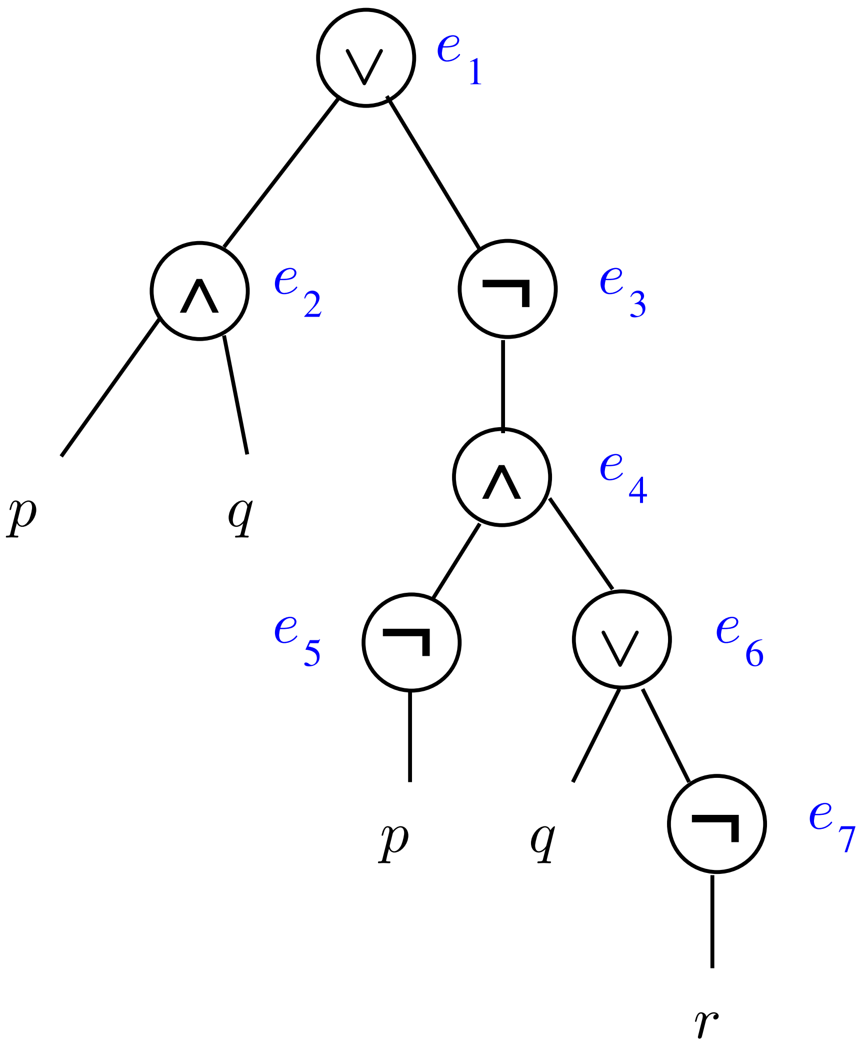

- e_1
- $e_1 \leftrightarrow e_2 \vee e_3$
- $e_2 \leftrightarrow p \wedge q$
- $e_3 \leftrightarrow \neg e_4$
- $e_4 \leftrightarrow e_5 \wedge e_6$
- $e_5 \leftrightarrow \neg p$
- $e_6 \leftrightarrow q \vee \neg e_7$
- $e_7 \leftrightarrow \neg r$

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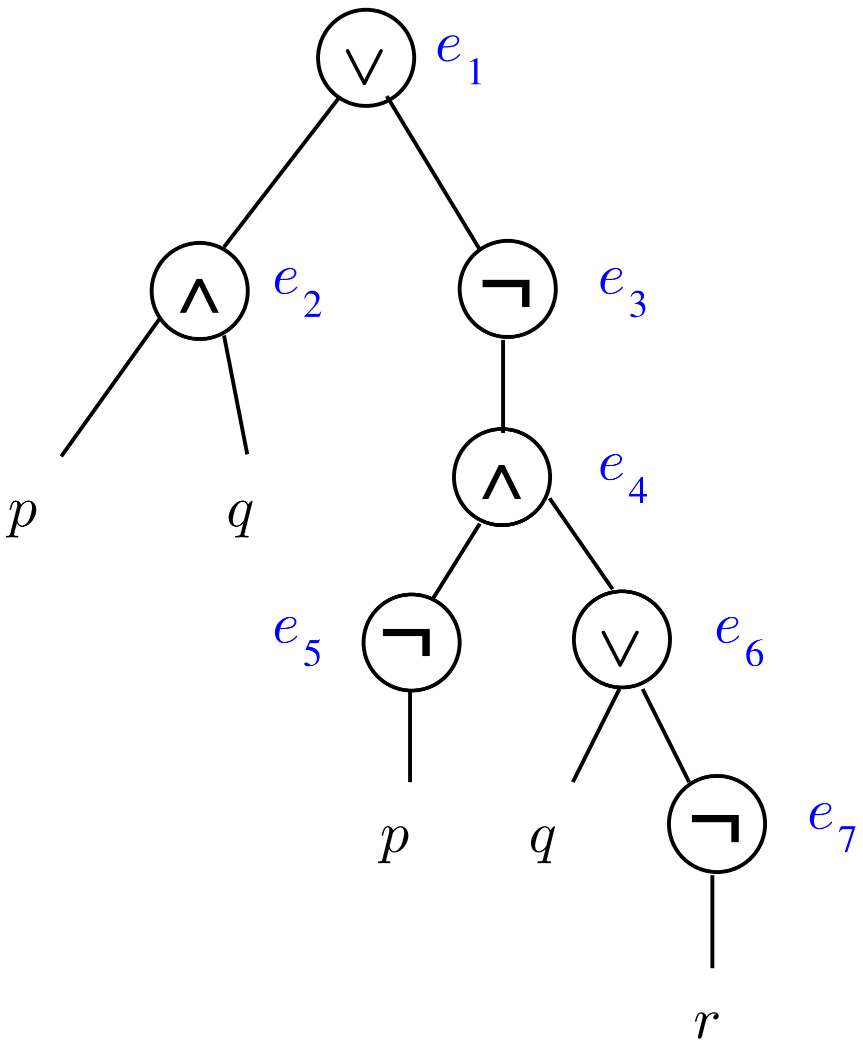

- e_1
- $e_1 \leftrightarrow e_2 \vee e_3$
  $\neg e_1 \vee e_2 \vee e_3$
  $\neg e_2 \vee e_1$
  $\neg e_3 \vee e_1$
- $e_2 \leftrightarrow p \wedge q$
- $e_3 \leftrightarrow \neg e_4$
- $e_4 \leftrightarrow e_5 \wedge e_6$
- $e_5 \leftrightarrow \neg p$
- $e_6 \leftrightarrow q \vee \neg e_7$
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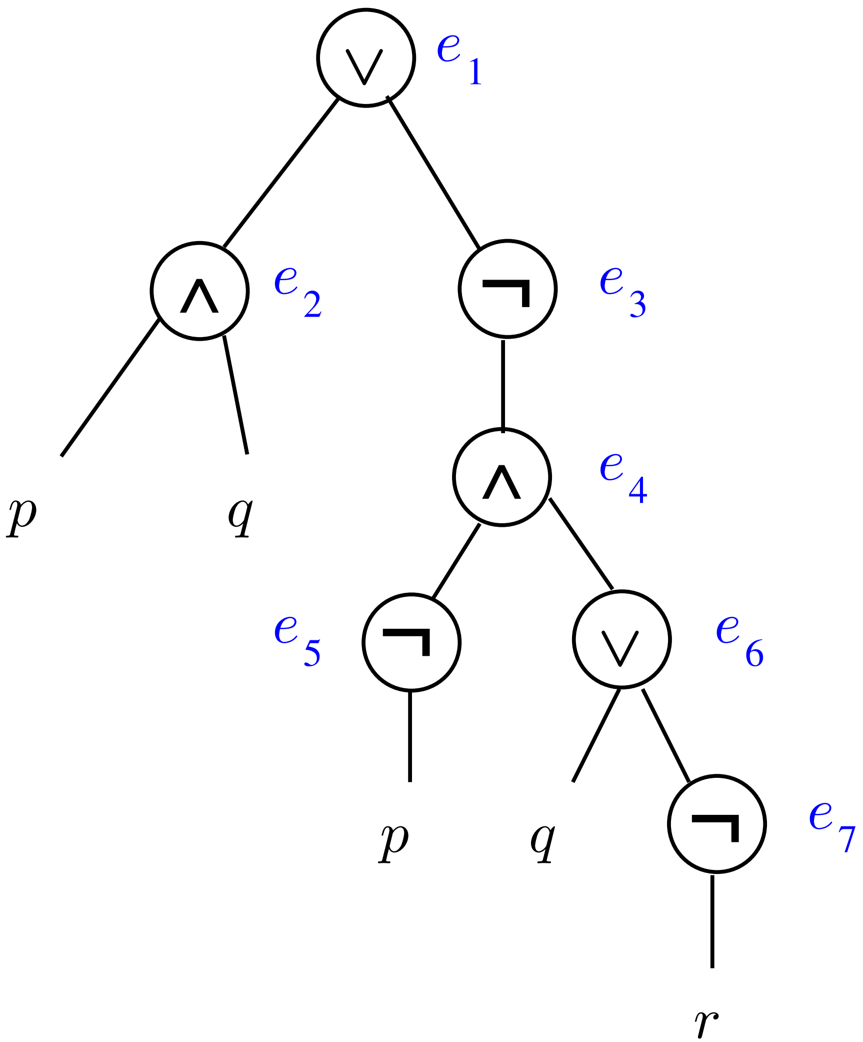


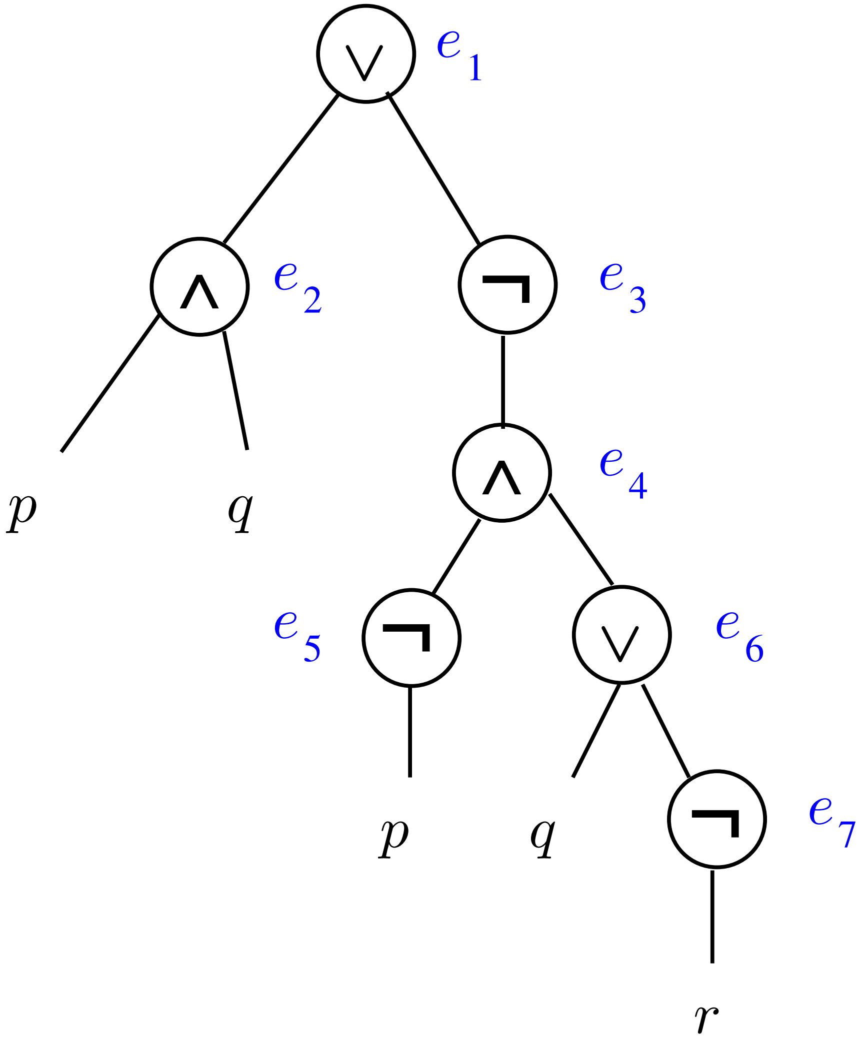
- e_1
- $e_1 \leftrightarrow e_2 \vee e_3$

  $\neg e_1 \vee e_2 \vee e_3$
  $\neg e_2 \vee e_1$
  $\neg e_3 \vee e_1$
- $e_2 \leftrightarrow p \wedge q$

  $\neg p \vee \neg q \vee e_2$
  $\neg e_2 \vee p$
  $\neg e_2 \vee q$
- $e_3 \leftrightarrow \neg e_4$
- $e_4 \leftrightarrow e_5 \wedge e_6$
- $e_5 \leftrightarrow \neg p$
- $e_6 \leftrightarrow q \vee \neg e_7$
- $e_7 \leftrightarrow \neg r$

Tseitin Transformation

Let F be $(p \wedge q) \vee \neg(\ \neg p \wedge (q \vee \neg r)\ )$



- e_1
- $e_1 \leftrightarrow e_2 \vee e_3$
  $\neg e_1 \vee e_2 \vee e_3$
  $\neg e_2 \vee e_1$
  $\neg e_3 \vee e_1$
- $e_2 \leftrightarrow p \wedge q$
  $\neg p \vee \neg q \vee e_2$
  $\neg e_2 \vee p$
  $\neg e_2 \vee q$
- $e_3 \leftrightarrow \neg e_4$
  $\neg e_3 \vee \neg e_4$
  $e_3 \vee e_4$
- $e_4 \leftrightarrow e_5 \wedge e_6$
- $e_5 \leftrightarrow \neg p$
- $e_6 \leftrightarrow q \vee \neg e_7$
- $e_7 \leftrightarrow \neg r$

Tseitin Transformation

- Variations of Tseitin transformation are the ones used in practice
- Tseitin transformation does not produce an equivalent CNF: for example, the Tseitin transformation of $F = \neg p$ is $G = e \wedge (\neg e \vee \neg p) \wedge (e \vee p)$, and

e	p	F	G
0	0	1	0
0	1	0	0
1	0	1	1
1	1	0	0

- Still, CNF obtained from F via Tseitin transformation has nice properties:
  - It is equisatisfiable to F
  - Any model of CNF projected to the variables in F gives a model of F
  - Any model of F can be completed to a model of the CNF
  - Can be computed in linear time in the size of F
- Hence no model is lost nor added in the transformation